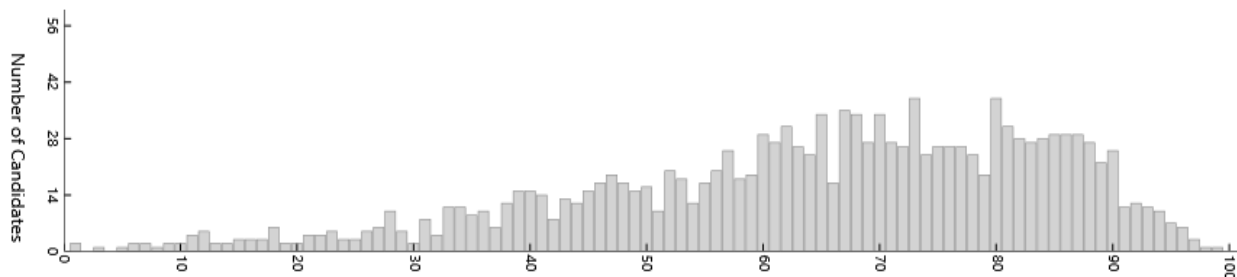




2016 ATAR course examination report: Mathematics Specialist

Year	Number who sat	Number of absentees
2016	1427	17

Examination score distribution



Summary

The examination had a mean of 64.13%. Candidate scores ranged from 0.66% to 97.66% with a standard deviation of 19.70%. The section means were: Section One: Calculator-free 73.61% with a standard deviation of 19.64%; and Section Two: Calculator-assumed 59.20% with a standard deviation of 20.34%.

Attempted by 1427 candidates

Mean 64.13%(/100) Max 99.33% Min 0.66%

Section means were:

Section One: Calculator-free

Mean 25.76(/35)

Max 35.00

Min 0.66

Section Two: Calculator-assumed

Mean 38.49(/65)

Max 64.33

Min 0.00

General comments

The paper proved to be very accessible, and from all reports, well received by candidates. Most candidates appeared to have sufficient time to answer all questions. There was a very large diversity in the standard of responses, with many candidates performing at a very high level while other candidates struggled. The Calculator-free questions were a little too straightforward as evidenced with a mean of 73.61%. The Calculator-assumed questions were deemed to be of an appropriate standard with a mean of 59.20%.

Sections of the course that were generally not understood well are listed below.

- In Question 1(b) of Section One, the ability to determine the domain and range of a composite function. This was surprising since this was not a new part of the course.
- Candidates' decision to use polar form versus Cartesian form for a complex number in Question 2(b). A significant number of candidates chose not to convert to polar form, to their peril.
- The use of a relevant trigonometric identity to integrate a function in Question 5(b).
- With Question 7(a) the vector equation of a sphere given the end points of the diameter, candidates were commonly unable to correctly determine the position vector for the centre.
- The use of the domain and range of a function in determining the defining rule for the inverse function in Question 8(d) was an area of concern.
- In Section Two, Question 11(a) and Question 15(a), the idea of showing a result is true from given information, found many candidates assuming what was meant to be obtained, or did not show how the result had been obtained.
- The concept of the slope field and the information contained in the given diagram was required for Question 18(a). This was from a new part of the course, and despite the

question being almost identical to that in the sample paper, candidates seemed ill-prepared.

- The use of a single sample mean to produce a confidence interval for an unknown population mean in Question 19(e) caused some consternation. The words 'significantly more' should have been interpreted in the context of making a reasoned comment. This new section of the course showed that candidates did not appear to understand what had to be done.

Advice for candidates

- Write legibly using a ball point pen, or at least a dark 2B pencil as using faint pencil does not scan particularly well.
- If an error is made using a pen, simply put a line through it.
- Show all your working particularly in questions where a CAS calculator routine has been used.
- When you are working in the statistics section, write mathematical statements, not language specific to a CAS calculator.
- Acknowledge a variable is normally distributed and show clearly the parameters (mean and standard deviation) used.
- When questions are worth three or more marks, do not simply write an answer as this will not attract full marks.

Advice for teachers

- Provide students with opportunities to explain ideas, using appropriate mathematics language.
- Ensure students understand the importance of the legibility of their work, the need to show all working, to write clear mathematics statements rather than language specific to a CAS calculator.
- Focus students' conceptual understanding on the new syllabus areas. Notably, the idea of using a sample mean to determine an interval for an unknown population mean, and then comparing this to the distribution of another variable.
- Interpretation of a diagram of a slope field needs emphasis.
- Provide opportunities for students to develop skills in showing a result is true from given information.

Comments on specific sections and questions

Section One: Calculator-free

Attempted by 1425 candidates

Mean 25.76(/35)

Max 35.00

Min 0.66

Candidates found this section to be very straightforward, with many candidates achieving a perfect score. Candidates performed well in the in the following areas:

- use of partial fractions to integrate a function (Question 4)
- standard techniques of integration using a change of variable (Question 5(a))
- knowledge of how a system of equations may have no solution (Question 6(b))
- graphing functions $y = \frac{1}{f(x)}$ and $y = f(|x|)$ (Question 8(a) and (b))
- solving an equation in the complex plane (Question 3(b)).

Question 1 attempted by 1425 candidates

Mean 2.43(/4)

Max 4.00

Min 0.00

Part (a) was done well. In part (b)(i) it was surprising that candidates typically could not state a correct domain. Many wrote that $x > 0$ (from the domain of $\ln(x)$) but did not consider that $x = 1$ would cause division by zero. A common error in part (b)(ii) was to think that the range comprised of all real numbers. A disappointing result overall for part (b).

Question 2 attempted by 1409 candidates Mean 5.34(/7) Max 7.00 Min 0.00

In part (a) some had problems in expanding $(1-i)^2$ and, for many candidates, assuming

that an answer like $\frac{1}{2} - \frac{1}{i}$ is of the form $a+bi$. The idea in part (b) was that candidates

would be expected to convert to polar form, apply DeMoivre's Theorem and then convert back to the form $a+bi$. Drawing a diagram would help to know whether the real/imaginary part is positive/negative. Those that did were hampered by a lack of knowledge of the exact trigonometric values for sine and cosine of $-\frac{5\pi}{6}$. Generally, those candidates who decided

to expand using the binomial theorem or successively used the distributive property, did not have the requisite algebra skills to obtain a correct answer.

Question 3 attempted by 1414 candidates Mean 4.48(/5) Max 5.00 Min 0.00

Part (a) was quite well done with a pleasing majority of candidates showing that the evaluation of $f(-4)$ was actually zero rather than just stating/assuming it. Part (b) was fairly

well done, finding the quadratic factor $z^2 - 2z + 3$. However, it was surprising how many candidates felt they had to resort to the quadratic formula rather than use the completion of square method, which led to errors.

Question 4 attempted by 1411 candidates Mean 5.55(/6) Max 6.00 Min 0.00

Part (a) was done well apart from arithmetic slips in solving for a, b . The use of partial fractions to integrate using the natural logarithmic function in part (b) was known well. A constant of integration was required for full marks, which pleasingly, most candidates included.

Question 5 attempted by 1409 candidates Mean 5.20(/7) Max 7.00 Min 0.00

The integration using the trigonometric substitution was handled well in part (a). A minority of candidates recognised the trigonometric identity $1 + \tan^2 x = \sec^2 x$ as being relevant in part (b). Candidates had to be careful to allow for the 'undoing' of the chain rule and multiply

by the factor $\frac{2}{\pi}$ in integrating $\sec^2\left(\frac{\pi x}{2}\right)$.

Question 6 attempted by 1418 candidates Mean 4.55(/6) Max 6.00 Min 0.00

Part (a) was done well, except for minor slips of algebra/arithmetic. In part (b) most candidates showed that they knew how the system of equations was going to result in no solution. There was evidence of good preparation by candidates.

Question 7 attempted by 1373 candidates Mean 4.48(/7) Max 7.00 Min 0.00

For what was supposed to be a routine question, it was disappointing in part (a) that a large number of candidates could not determine correctly the centre position vector, instead performing vector subtraction which was not relevant for determining the centre. Some candidates underlined their confusion in writing a dot product or writing a Cartesian equation. Part (b) was quite well done with most candidates choosing to form the cross product to find the normal vector for the plane. Hence, it appeared that the idea to use the cross product was understood. Errors in its calculation were common, often with the order of subtraction with components.

Question 8 attempted by 1417 candidates Mean 7.42(/11) Max 11.00 Min 0.00

The concept of drawing the graph of $y = \frac{1}{f(x)}$ was done well in part (a) and indicated that

candidates were well-prepared for this new element of the course. However, there was a lack of care/detail in graphs, despite having the overall features correct. Part (b) was also done well although many candidates drew the graph of $y = |f(x)|$. In part (c) many

candidates could read and understand what this question part required and could provide a coherent reason for why the domain restriction was needed. Others started to write the rule for $y = f^{-1}(x)$ rather than in the space for part (d). Some candidates appeared to be ill-prepared to write a brief sentence to accompany an answer. In part (d) virtually all candidates knew how to begin to write an inverse, but very few showed that they knew that the range of the inverse function has to be equal to the domain of the given function. Hence, the negative of the square root had to be considered.

Section Two: Calculator-Assumed

Attempted by 1424 candidates

Mean 38.49(/65)

Max 64.33

Min 0.00

Candidates found this section to be more challenging than the Calculator-free section, providing opportunities for the more able candidates to show their capabilities. Most candidates seemed to have had time to answer the vast majority of the questions, including attempting the last question on the paper. Candidates performed well in the in the following areas:

- determining the speed of a particle using vectors (Question16(b))
- determining the size of a sample to achieve a given difference from a population mean (Question 19(d))
- finding the position vector for the intersection of a line and a plane (Question 20(a))
- integration using a change of variable (Question 9)
- solving an equation in the complex plane (Question14(a)).

Question 9 attempted by 1393 candidates

Mean 2.99(/5)

Max 5.00

Min 0.00

Most candidates entered into the spirit of the question well using the change of variable and writing an anti-derivative. Unfortunately, many were let down by their index laws skills, often making the indices simpler. Predictably, some candidates reached for their CAS calculator, and for those that did so without showing the due processes required, scored either 1 or 2 marks out of 5.

Question 10 attempted by 1409 candidates

Mean 5.67(/9)

Max 9.00

Min 0.00

The circle locus was recognised by almost all candidates. Part (b) was quite well done, with the expected errors of shading the incorrect region, identifying the incorrect key points or, for many using faint pencil, no shading was visible at all. Part (c) was a more conceptual question using the idea that a complex number can be viewed as a vector. Those candidates who gave a correct answer without any supporting evidence did not receive full marks. A good way to indicate the evidence was to draw the position of the maximum modulus on the diagram from part (a) or as some candidates did, draw another diagram showing the key elements.

Question 11 attempted by 1400 candidates

Mean 5.07(/7)

Max 7.00

Min 0.00

Candidates needed to start with the acceleration function $a(t) = kt$ in part (a), then develop the velocity function as an anti-derivative and use the information that $v(0) = 0$ and

$v(4) = 1.2$ to deduce the value of k . Assuming what has to be shown (in general) is not a sound way to begin and candidates should start with what is given. Part (b) was done well with the exception of some confusion between which function derivative had to be evaluated to find the approximate change. Part (c) was a reasonably straightforward question that proved to be elusive for many. The main error was in assuming that the lift was only ascending for the first four seconds, rather than the entire 20 seconds.

Question 12 attempted by 1399 candidates

Mean 3.88(/6)

Max 6.00

Min 0.00

Part (a) was answered well by almost all candidates, with the common error stating that $a = 2$. Part (b) was best answered by having a conceptual understanding or being assisted

by drawing a graph of $y = |f(x)|$ and seeing how a horizontal line can intersect in four positions. Those that wanted to write algebraic equations with absolute value appeared to be prepared for a question that did not equate with the current syllabus. Many candidates did not appear to be prepared to explain an answer.

Question 13 attempted by 1408 candidates Mean 4.22(/5) Max 5.00 Min 0.00
Part (a) was a straightforward use of implicit differentiation. Some wished to complicate matters by writing inverse trigonometric functions and then by differentiating, sometimes successfully. Routine application to find the area bounded by a curve and a line was required in part (b). An error with the upper limit of integration was common. Some candidates calculated a volume.

Question 14 attempted by 1400 candidates Mean 4.89(/7) Max 7.00 Min 0.00
Part (a) was a routine complex equation to be solved using DeMoivre's Theorem. Errors principally, were in finding the correct argument for $-16i$. Drawing a diagram should have convinced candidates that this was $-\frac{\pi}{2}$. Almost all candidates knew that the roots were equally separated. Confusion reigned with the wording 'least positive' argument in part (b). Allowance was made for the interpretation 'most negative'. A CAS calculator approach would have made this a simple matter, but many candidates thought that the 'universal distributive law' applied i.e. $\arg(w+2) = \arg(w) + 2$.

Question 15 attempted by 1398 candidates Mean 6.19(/10) Max 10.00 Min 0.00
Candidates had significant difficulty with part (a) in deriving the expression for the area of the triangular base when the depth was h i.e. they could not perceive the necessary trigonometry to relate side lengths. Many tried to unsuccessfully 'fudge' their attempts. At this level, one would expect candidates to be able to show/derive a result such as this. In part (b) most candidates understood that this question tested related rates. However, in many cases, the negative value for the volume rate was overlooked. Pleasingly, most gave the correct units of measurement for the rate answer. Part (c) was fairly straightforward as the generalisation of part (b). This question was given to assist candidates in leading them into part (d). The separation of variables technique was applied well in part (d) but candidates had problems finding the correct constant(s) of integration. The initial depth was not given, thus candidates had to find this themselves, which created difficulties.

Question 16 attempted by 1342 candidates Mean 4.55(/10) Max 10.00 Min 0.00
Part (a) was a disappointing performance in transforming the two parametric equations into a single Cartesian equation. The cosine double angle result was often applied poorly, with lack of brackets causing errors with distributive multiplication. A mere handful of candidates realised that a domain or range restriction was necessary to give the path as shown in the diagram, and not the entire parabola. Part (b) showed a very good set of responses of finding the speed of the particle. Candidates seemed to be prepared well for this question.

Errors occurred mainly in solving incorrectly for time or assuming that $\frac{dy}{dx}$ was the

determinant for speed. The majority of candidates in part (c) did not know how to write an expression to determine the distance travelled. The limits of integration were often incorrect or ignored, with a small number able to achieve full marks. Poor notation was rife, albeit not penalised.

Question 17 attempted by 1373 candidates Mean 5.44(/8) Max 8.00 Min 0.00
Overall part (a) was done quite well. The majority of candidates were able to differentiate implicitly to determine the gradient of the tangent. The alternative in using the value of the radial gradient to determine the tangent gradient (since the two formed a right angle) was accepted provided there was supporting evidence. Part (b) was straightforward for candidates. Part (c) of the question was a reasonably straightforward volume question, yet the performance overall was very mixed. Errors with incorrect integrands, incorrect limits of

integration and the inability to use a CAS calculator to evaluate correctly were common. The incorrect limits suggested many candidates did not comprehend the region that was rotated.

Question 18 attempted by 1367 candidates Mean 3.27(/7) Max 7.00 Min 0.00

It was anticipated that candidates in part (a) would perform well given that the question was almost identical to the corresponding slope field question on the sample paper circulated to schools previously. This assumption proved false, with significant confusion regarding the question requirements. A general equation was required, rather than a specific rule, but with some indication that in writing a linear rule, the vertical intercept was positive and the gradient was negative. Candidates performed slightly better in part (b). It was a reasonable expectation that candidates observed that the slope field had a value of zero at $x = 2$ (from the graph's horizontal isoclines), along with using the given information that the slope field had a value of 0.5 at $x = 1$. Many candidates had difficulty understanding this new section of the syllabus.

Question 19 attempted by 1398 candidates Mean 9.84(/16) Max 16.0 Min 0.00

Part (a) was a straightforward question although candidates need to be instructed to state that the sample mean is normally distributed, and make a clear statement about its parameters (the mean and standard deviation). Far too many express themselves poorly/minimally, often resorting to CAS-speak type language instead of recognisable mathematics statements. The same comment applies to part (b) with many candidates not writing the event $\bar{W} < 179.2$. Candidates should be aware that if a question is worth three marks, more than an answer is required for full marks. Part (c) was reasonably well done, with most candidates able to indicate the correct z -score required for 99% confidence. Part (d) was very well answered. Candidates seemed well prepared for this question. Strictly speaking, an inequality should be written since the question stated that the difference from the population mean had to be less than five litres. Part (e) tested whether candidates knew what the term 'significantly more' meant in the given context. It offered candidates the opportunity to show their understanding of what it means to form a confidence interval for an unknown population mean. In this case, the WolliWorks population mean is not known, and the question is essentially whether the WolliWorks population mean is significantly larger than the SavaDaWater population mean. Many candidates showed their confusion in using the SavaDaWater population mean of 175 to obtain an interval for Wolliworks water usage. This new element of the syllabus was not understood and will require attention.

Question 20 attempted by 1231 candidates Mean 3.14(/7) Max 7.00 Min 0.00

A good set of responses in finding the intersection between a line and a plane in part (a). Despite this, notation was poor or lacking in answers. Despite part (b) being the most challenging question on the paper, more than 60% of the cohort attempted it, albeit not exhibiting the desired behaviours to score marks. This question provided many avenues to a correct solution, ultimately by solving simultaneous equations using a CAS calculator. The best candidates were able to score full marks, proving it to be a discriminator. Some candidates provided some original and elegant responses to this question.